Orientifold Limit of F-Theory Vacua*

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February 1, 2008

Abstract

We show how F-theory on a Calabi-Yau $(n+1)$-fold, in appropriate limit, can be identified as an orientifold of type IIB string theory compactified on a Calabi-Yau n-fold.

hep-th/9709159
MRI/PHY/P970924

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*Talk given at the Trieste conference on Duality Symmetries and at Strings 97

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Orientifolds and F-theory are two apparently different ways of compactifying type IIB string theory. In this talk we shall explore the relationship between these two different classes of type IIB compactification. In particular, we shall show how F-theory on a Calabi-Yau $(n+1)$-fold, in appropriate limit, reduces to an orientifold of type IIB string theory compactified on a Calabi-Yau n-fold. This talk will be based mainly on ref.[1] and also partially on refs.[2, 3]. All other relevant references can be found in these papers.

We begin with some facts about type IIB string theory. Massless bosonic fields in type IIB theory come from two sectors. The Neveu-Schwarz $-$ Neveu-Schwarz (NS) sector contributes the graviton $G_{\mu\nu}$, the anti-symmetric tensor field $B_{\mu\nu}$, and the dilaton Φ. The Ramond-Ramond (RR) sector contributes another scalar a, sometimes called the axion, another anti-symmetric tensor field $B'_{\mu\nu}$, and a rank four anti-symmetric tensor field $D_{\mu\nu\rho\sigma}$ with self-dual field strength. We define

$$\lambda \equiv a + i e^{-\Phi} \,. \tag{1}$$

This theory has two perturbatively realised Z_2 symmetries. The first one $-$ denoted by $(-1)^{F_L}$ where F_L is the contribution to the space-time fermion number from the left moving sector of the world sheet $-$ changes sign of a, $B'_{\mu\nu}$, $D_{\mu\nu\rho\sigma}$, leaving the other massless bosonic fields unchanged. The second one $-$ the world-sheet parity transformation Ω $-$ changes the sign of $B_{\mu\nu}$, a and $D_{\mu\nu\rho\sigma}$. Besides these two symmetries which are valid order by order in perturbation theory, this theory also has a conjectured non-perturbative symmetry[4], known as S-duality, under which

$$\lambda \to \frac{p\lambda + q}{r\lambda + s}, \qquad \begin{pmatrix} B_{\mu\nu} \\ B'_{\mu\nu} \end{pmatrix} \to \begin{pmatrix} p & q \\ r & s \end{pmatrix} \begin{pmatrix} B_{\mu\nu} \\ B'_{\mu\nu} \end{pmatrix} . \tag{2}$$

Here p, q, r, s are integers satisfying $ps - qr = 1$. We shall denote by S and T the following specific SL(2,Z) transformations:

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} , \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} . \tag{3}$$

Studying the action of the various transformations on the massless fields, we can identify the discrete symmetry transformation $(-1)^{F_L} \cdot \Omega$ with the SL(2,Z) transformation:

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{4}$$

Orientifolds are orbifolds of (compactified) type IIB theory, where the orbifolding group involves the world-sheet parity transformation Ω. In this talk we shall focus our attention on a class of orientifolds of type IIB on Calabi-Yau manifolds defined as follows. Let $\mathcal{M}_n$ be a Calabi-Yau n-fold (n complex dimensional manifold), and σ be a Z_2 symmetry of $\mathcal{M}_n$ such that

- it reverses the sign of the holomorphic n-form on $\mathcal{M}_n$ and,
- its fixed point sets form a submanifold of complex codimension 1.

We now consider the orientifold:

$$IIB \quad \text{on} \quad \mathcal{M}_n \times R^{9-2n,1}/(-1)^{F_L} \cdot \Omega \cdot \sigma\,, \tag{5}$$

where $R^{9-2n,1}$ denotes the $(10-2n)$ dimensional Minkowski space. The transformation $(-1)^{F_L} \cdot \Omega \cdot \sigma$ preserves half of the space-time supersymmetry. The fixed point set on $\mathcal{M}_n \times R^{9-2n,1}$ under σ is of real codimension two in the (9+1) dimensional space-time, and are known as orientifold seven planes. This is known to carry -4 units of RR charge. Thus the SL(2,Z) monodromy along a closed contour around this orientifold plane is $-T^{-4}$. Here -1 is the effect of the monodromy $(-1)^{F_L} \cdot \Omega$, and T^{-4} is the effect of RR charge carried by the orientifold plane. Since there is no non-compact direction transverse to the orientifold plane, this monodromy must be cancelled by placing Dirichlet 7-branes (D7-branes) along appropriate subspaces of complex codimension one in $\mathcal{M}_n \times R^{9-2n,1}$[5]. Since the D7-branes carry 1 unit of RR charge, the monodromy around a D7-brane is T. We must place these D-branes in such a way that for any S^2 embedded in $\mathcal{M}_n$, the total monodromy around all the points where this S^2 intersects the orientifold plane and the D-branes vanish (see Fig.1). Also, the D-brane configuration must be such so as not to break any further supersymmetry. We shall see later how this can be achieved in practice.

Let us now turn to a brief review of F-theory[6]. The starting point of an F-theory compactification is an elliptically fibered (fibers are two dimensional tori) Calabi-Yau $(n+1)$-fold $\mathcal{M}_{n+1}$ on a base $\mathcal{B}_n$. Let $\vec{u}$ be the complex coordinates on $\mathcal{B}_n$, and $\tau(\vec{u})$ denote the modular parameter of the fiber T^2 as a function of $\vec{u}$. By definition, F-theory compactified on $\mathcal{M}_{n+1}$ is type IIB on $\mathcal{B}_n \times R^{9-2n,1}$ with

$$\lambda(\vec{u}) = \tau(\vec{u})\,. \tag{6}$$

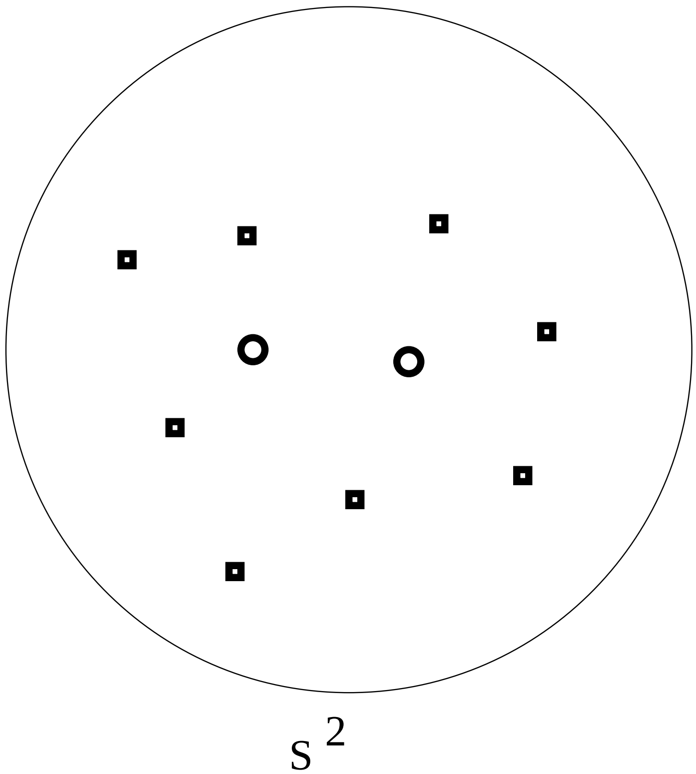

Figure 1: In this figure we have displayed a two dimensional sphere S^2 embedded in $\mathcal{M}_n$. The black squares represent points where the D-branes intersect this sphere, and the black circles represent points where the orientifold planes intersect the sphere. The monodromy around a curve enclosing all the squares and the circles must be trivial since this curve can be contracted to a point in S^2. This forces the number of circles to be even and the number of squares to be four times the number of circles.

It will be useful to consider the Weierstrass form of elliptically fibered manifold:

$$y^2 = x^3 + f(\vec{u})x + g(\vec{u})\,, \tag{7}$$

where x, y are complex variables, and $f(\vec{u})$, $g(\vec{u})$ are sections of appropriate line bundles on $\mathcal{B}_n$. In particular, we shall choose f and g to be sections of $L^{\otimes 4}$ and $L^{\otimes 6}$ respectively, where $L^{\otimes n}$ denote the nth power of some line

bundle L. We can then make sense of eq.(7) by regarding x and y as elements of $L^{\otimes 2}$ and $L^{\otimes 3}$ respectively. For every $\vec{u}$ we have a torus labelled by (x, y) satisfying (7), with modular parameter τ given by:

$$j(\tau) = 4 \cdot (24f)^3/(4f^3 + 27g^2)\,. \tag{8}$$

j is the modular function with a single pole at $i\infty$. F-theory on this elliptically fibered manifold is type IIB on $\mathcal{B}_n$ with

$$j(\lambda(\vec{u})) = 4 \cdot (24f)^3/(4f^3 + 27g^2)\,. \tag{9}$$

$j(\lambda) \to \infty$ at zeroes of

$$\Delta \equiv (4f^3 + 27g^2)\,. \tag{10}$$

At these points $\lambda \to i\infty$ *up to SL(2,Z) transformation*. These are surfaces of complex codimension one, and are known as the locations of the seven branes, although, as we shall see soon, they are not necessarily Dirichlet seven branes. Monodromy around each of these seven branes is conjugate to SL(2,Z) transformation T.

We shall now take an appropriate 'weak coupling limit' such that the monodromy around the zeroes of Δ look identical to that of an orientifold. For this, let us take:

$$\begin{aligned} f &= -3h^2 + C\eta \\ g &= -2h^3 + Ch\eta + C^2\chi\,. \end{aligned} \tag{11}$$

Here C is a constant, and h, η, χ are sections of line bundles $L^{\otimes 2}$, $L^{\otimes 4}$ and $L^{\otimes 6}$ respectively. There is no loss of generality in the choice of f and g given in (11), since for fixed h and C, we can vary η and χ to span the whole range of f and g. On the other hand, there is clearly a redundancy in this choice, since C and h could be absorbed in η and χ. Put another way, for a given f and g, we can choose C, η, h and χ in many different ways. Nevertheless we shall keep this redundancy, as this will help us take the weak coupling limit properly.

With the above representation of f and g, we get

$$\begin{aligned} \Delta &= (4f^3 + 27g^2) \\ &= C^2\{\eta^2(4C\eta - 9h^2) + 54h(C\eta - 2h^2)\chi + 27C^2\chi^2\}\,, \end{aligned} \tag{12}$$

$$j(\lambda) = 4 \cdot (24)^3 \cdot (C\eta - 3h^2)^3/\Delta \,. \tag{13}$$

Now we take the 'weak coupling limit' $C \to 0$. In this limit

$$\Delta \simeq C^2(-9h^2)(\eta^2 + 12h\chi) \,. \tag{14}$$

Thus the zeroes of Δ are at

$$h = 0, \qquad \text{and} \qquad \eta^2 + 12h\chi = 0 \,. \tag{15}$$

Also, $j(\lambda)$ is large everywhere on the base except in regions

$$|h| \sim |C|^{1/2} \,. \tag{16}$$

Let us now recall that large $j(\lambda)$ corresponds to large $Im(\lambda)$ up to an SL(2,Z) transformation. Thus, at every point in the region $|h| >> |C|^{1/2}$, either $Im(\lambda)$ is large, or λ approaches a rational point on the real axis. Since $h = 0$ corresponds to a surface of real codimension two, the region $h \neq 0$ is connected. Thus for small $|C|$, the region $|h| >> |C|^{1/2}$, where $j(\lambda)$ is large, is also connected (the unshaded region in Fig. 2). This shows that if we choose our SL(2,Z) convention in such a way that $Im(\lambda)$ is large at one point in the region $|h| >> |C|^{1/2}$ (which can always be done), then it must remain large in the whole of this region. This is the way we shall choose our convention from now on. Since large $Im(\lambda)$ corresponds to weak coupling, we see that in this convention, the $C \to 0$ limit corresponds to coupling constant being small in most part of the base. It is in this sense that $C \to 0$ represents weak coupling limit.

We shall now compute the monodromy around the zeroes of Δ given in (14). In doing this computation, we should keep in mind that as long as we choose a contour avoiding the $|h| \sim |C|^{1/2}$ regions (*e.g.* C_1 or C_2 in Fig.2), $Im(\lambda)$ is large all along the contour, and hence the only allowed SL(2,Z) monodromy along such a contour is $\pm T^n$. First we shall focus on the monodromies around the 7-branes at

$$\eta^2 + 12h\chi = 0 \,. \tag{17}$$

Unless this expression has an accidental double zero, this represents single zeroes of Δ. Thus monodromy around such a singularity must be SL(2,Z)

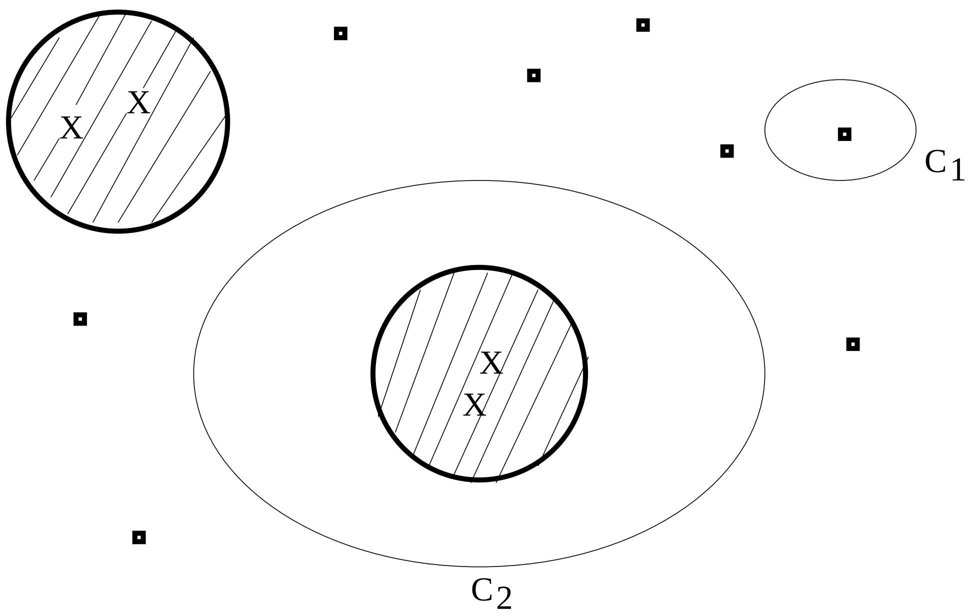

Figure 2: This figure displays a two dimensional section of $\mathcal{B}_n$. The black squares represent the zeroes of $(\eta^2 + 12h\chi)$. The shaded regions denote the region $|h| \sim |C|^{1/2}$, and the two black crosses inside each of the shaded region are the two zeroes of Δ near $h = 0$. A contour around only one of these black crosses must pass through the shaded region, whereas a contour around both crosses can avoid the shaded region. C_2 denotes such a contour. C_1 is a contour around a zero of $(\eta^2 + 12h\chi)$. In the unshaded region of this diagram, $Im(\lambda)$ is large. As shown in the text, under F-theory $-$ orientifold correspondence, each shaded region is mapped to a black circle in Fig.1.

conjugate to T. We can choose the contour around this hyperplane keeping it away from the $|h| \sim |C|^{1/2}$ region (*e.g.* C_1 in Fig.2). Hence, the monodromy must be of the form $\pm T^n$. Combining these two requirements, we see that the monodromy must be T. Since this is the monodromy around a D7-brane, we see that for small $|C|$, (17) represent the locations of Dirichlet 7-branes.

Next we turn to analyze the monodromy around the hypersurface

$$h = 0\,. \tag{18}$$

Note that Δ has double zeroes at these locations. However, this double

zero appears only as $C \to 0$. For small non-zero C, Δ will have a pair of zeroes around $h = 0$ as displayed in Fig.2. The monodromies around these individual zeroes are conjugate to T. Let us take these monodromies to be

$$MTM^{-1} \qquad \text{and} \qquad NTN^{-1}\,, \tag{19}$$

respectively. Here M and N are two SL(2,Z) matrices. Then the total monodromy around the $h = 0$ surface is

$$MTM^{-1}NTN^{-1}\,. \tag{20}$$

Since the contour around the individual zeroes of Δ around $h = 0$ must pass through the region $|h| \sim |C|^{1/2}$, we cannot conclude that these monodromies must be of the form $\pm T^n$ (see Fig. 2). However, a contour C_2 encircling both these zeroes can be taken to be away from this region. Hence (20) must be of the form $\pm T^n$. We shall now try to determine the value of n, as well as the overall sign. For this, note that for large $Im(\lambda)$,

$$j(\lambda) \sim e^{-2\pi i\lambda}\,. \tag{21}$$

Thus

$$n \equiv \oint_{C_2} d\lambda = -\frac{1}{2\pi i}\oint_{C_2} d\ln j(\lambda)\,. \tag{22}$$

Thus in order to calculate n, we need to calculate the change in $\ln j(\lambda)$ as we go once around the contour. For small $|C|$ and $|h| >> |C|^{1/2}$, we have

$$j(\lambda) \sim h^4/C^2(\eta^2 + 12h\chi)\,. \tag{23}$$

Thus along a contour C_2 around $h = 0$, $\ln j(\lambda)$ changes by $4 \cdot 2\pi i$. This gives, from (22), $n = -4$. Hence the monodromy along the contour C_2 is $\pm T^{-4}$.

Next we turn to the determination of the overall sign. For this, recall that this monodromy must be expressible as $MTM^{-1}NTN^{-1}$ *i.e.* we need

$$MTM^{-1}NTN^{-1} = \pm T^{-4}\,. \tag{24}$$

It turns out that the most general solution of this equation is:

$$MTM^{-1} = \begin{pmatrix} 1-p & p^2 \\ -1 & 1+p \end{pmatrix}, \qquad NTN^{-1} = \begin{pmatrix} -1-p & (p+2)^2 \\ -1 & 3+p \end{pmatrix}, \tag{25}$$

giving

$$MTM^{-1}NTN^{-1} = -T^{-4}\,. \tag{26}$$

Here p is an arbitrary integer. This shows that the monodromy around $h = 0$ is $-T^{-4}$. In other words, for small C, the $h = 0$ plane behaves like an orientifold plane!

Thus we see that in the $C \to 0$ limit, the F-theory background can be identified to that of an orientifold with,

1. Orientifold 7-planes at $h(\vec{u}) = 0$, and

2. Dirichlet 7-branes at $\eta(\vec{u})^2 + 12h(\vec{u})\chi(\vec{u}) = 0$.

This analysis also shows that for small but finite C, the orientifold plane splits into two seven branes lying close to the surface $h = 0$. This reflects a phenomenon already observed earlier in a much simpler situation − namely orientifold of type IIB compactified on a two dimensional torus[7].

We would also like to find the original manifold whose orientifold this theory is. It is clear that this manifold must be a double cover of the base $\mathcal{B}_n$, branched along the orientifold plane $h(\vec{u}) = 0$. Let us now consider the manifold:

$$\mathcal{M}_n : \quad \xi^2 = h(\vec{u})\,, \tag{27}$$

where ξ is an element of the line bundle L. This manifold has a Z_2 isometry

$$\sigma : \qquad \xi \to -\xi\,. \tag{28}$$

Fixed point set under this isometry corresponds to the complex codimension one submanifold $\xi = 0$. Using eq.(27), this gives $h(\vec{u}) = 0$. Since for every point $\vec{u}$ in $\mathcal{B}_n$, except those at $h(\vec{u}) = 0$, we have two points in $\mathcal{M}_n$ given by $(\vec{u}, \xi = \pm\sqrt{h(\vec{u})})$, $\mathcal{M}_n$ is a double cover of $\mathcal{B}_n$ branched along $h(\vec{u}) = 0$. Thus $\mathcal{B}_n$ can be identified with $\mathcal{M}_n/\sigma$. This, in turn, shows that the precise description of the orientifold that we have found is

$$\text{Type IIB on } \mathcal{M}_n \times R^{9-2n,1}/(-1)^{F_L} \cdot \Omega \cdot \sigma\,. \tag{29}$$

The next question that arises naturally is: does $\mathcal{M}_n$ represent a Calabi-Yau n-fold? To answer this question, let us consider the original Calabi-Yau manifold $\mathcal{M}_{n+1}$ described in eq.(7) with f and g being sections of $L^{\otimes 4}$ and $L^{\otimes 6}$ respectively, and x and y being coordinates on $L^{\otimes 2}$ and $L^{\otimes 3}$ respectively.

In order that $\mathcal{M}_{n+1}$ is Calabi-Yau, we need its first Chern class to vanish. This imposes the following restriction on L:

$$c_1(\mathcal{B}_n) + c_1(L)(3+2-6) = 0\,. \tag{30}$$

In the coefficient of $c_1(L)$ the factors 3 and 2 represent the fact that y and x are coordinates on $L^{\otimes 3}$ and $L^{\otimes 2}$ respectively, whereas the factor 6 represents that the constraint (7) belongs to $L^{\otimes 6}$. On the other hand, since h is a section of $L^{\otimes 2}$, and ξ is a coordinate on L, in order that the auxiliary manifold $\mathcal{M}_n$ described in (27) is a Calabi-Yau manifold, we must have:

$$c_1(\mathcal{B}_n) + c_1(L)(1-2) = 0\,. \tag{31}$$

But this is identical to the condition (30) for $\mathcal{M}_{n+1}$ to be Calabi-Yau. Thus we see that $\mathcal{M}_n$ also describes a Calabi-Yau manifold, provided it is a non-singular manifold.

This finishes the outline of the general procedure by which we can map an F-theory compactification to an orientifold of type IIB in appropriate weak coupling limit. We shall now illustrate this by means of a few examples. The first example we shall consider will be F-theory on Calabi-Yau 3-fold on base $CP^1\times CP^1$. Let u, v be the affine coordinates on $CP^1\times CP^1$. An elliptically fibered Calabi-Yau 3-fold corresponds to choosing $f(u,v)$ to be a polynomial of degree (8,8) in (u,v) and $g(u,v)$ to be a polynomial of degree (12,12) in (u,v) in eq.(7). Then $h(u,v)$, $\eta(u,v)$ and $\chi(u,v)$ are respectively polynomials of degree (4,4), (8,8) and (12,12) in (u,v). According to our analysis, in the weak coupling limit this describes an orientifold

$$\text{Type IIB on } \mathcal{M}_2\times R^{5,1}/(-1)^{F_L}\cdot\Omega\cdot\sigma$$

where $\mathcal{M}_2$ corresponds to the manifold

$$\mathcal{M}_2: \qquad \xi^2 = h(u,v)\,. \tag{32}$$

This corresponds to a K3 surface. It can be shown that σ $(\xi\to -\xi)$ describes a Nikulin involution[9] on this surface with

$$(r,a,\delta) = (2,2,0)\,. \tag{33}$$

The D-branes are situated at

$$\eta(u,v)^2 + 12h(u,v)\chi(u,v) = 0\,. \tag{34}$$

We can simplify this model further by going to the orbifold limit of this K3. For this, we choose:

$$
\begin{aligned}
h(u,v) &= \prod_{\alpha=1}^{4}(u-\widetilde{u}_\alpha)(v-\widetilde{v}_\alpha) \\
\eta(u,v) &= \prod_{i=1}^{8}(u-u_i)(v-v_i), \qquad \chi = 0\,.
\end{aligned}
\tag{35}
$$

Here $\widetilde{u}_\alpha$, $\widetilde{v}_\alpha$, u_i, v_i are constants. This gives the following defining equation for $\mathcal{M}_2$:

$$
\mathcal{M}_2: \quad \xi^2 = \prod_{\alpha=1}^{4}(u-\widetilde{u}_\alpha)(v-\widetilde{v}_\alpha)\,. \tag{36}
$$

This corresponds to the T^4/Z_2 orbifold limit of K3. The $D7$-branes are now located in pairs at:

$$
u = u_i \qquad \text{and at} \qquad v = v_i. \tag{37}
$$

This model can be shown[2] to be T-dual to the Gimon-Polchinski model[8, 5].

By using similar methods one can show that F-theory on Calabi-Yau 3-folds on base F_1 and F_4 can be mapped to type IIB on $K3\times R^{5,1}/(-1)^{F_L}\cdot\Omega\cdot\sigma$ where σ corresponds to the Nikulin involution:

$$
\begin{aligned}
(r,a,\delta) &= (2,2,1) \qquad \text{for} \qquad \text{base} \quad F_1 \\
(r,a,\delta) &= (2,0,0) \qquad \text{for} \qquad \text{base} \quad F_4\,.
\end{aligned}
\tag{38}
$$

Using mirror symmetry, one can further relate type IIB on $K3/(-1)^{F_L}\cdot\Omega\cdot\sigma$ to type I on a mirror $K3$[3] which we shall denote by $K3'$. The three different choices of σ correspond to three different choices of gauge bundles for type I on $K3'$. In order to give a more specific description of this correspondence, let us take a two sphere C inside $K3'$, and let C_N and C_S denote the northern and southern hemispheres of C. Also let g_N and g_S denote the holonomies along the boundaries of C_N and C_S respectively in the *vector representation* of SO(32). Then $g_N g_S^{-1}$ is unity when regarded as an element of the group $Spin(32)/Z_2$, but not necessarily as an element of the group $SO(32)$. We define $\widetilde{w}_2$ to be an element of the second homology class of $K3'$ such that:

$$
g_N g_S^{-1} = \exp\left(i\pi(\widetilde{w}_2\cap C)\right), \tag{39}
$$

where $(\widetilde{w}_2 \cap C)$ denotes the intersection number of $\widetilde{w}_2$ and C. Then the gauge bundle of type I on $K3'$ can belong to either of the three classes characterized by the following properties on $\widetilde{w}_2$[10, 11]:

- $\widetilde{w}_2 = 0$,
- $(\widetilde{w}_2 \cap \widetilde{w}_2) = 0 \quad mod \quad 4$,
- $(\widetilde{w}_2 \cap \widetilde{w}_2) = 2 \quad mod \quad 4$.

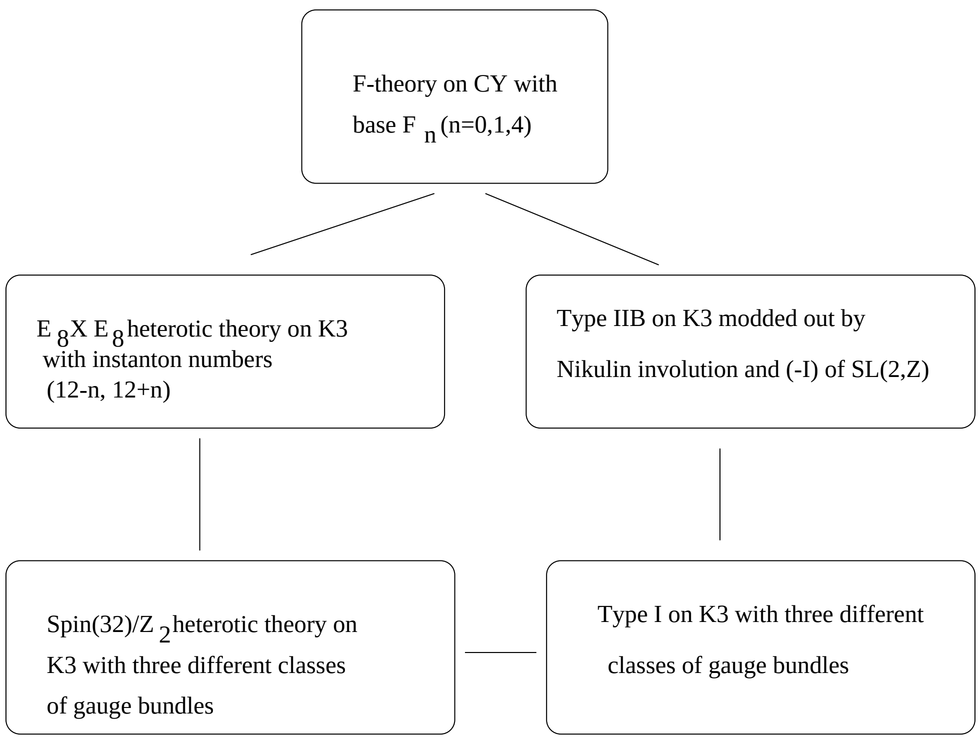

Figure 3: The Duality Cycle

One can show that[3] these three different classes of type I compactifications are related, by a mirror transformation, to type IIB on $(K3 \times R^{5,1}/(-1)^{F_L} \cdot \Omega \cdot \sigma)$ where σ is a Nikulin involution with

- $(r, a, \delta) = (2, 0, 0)$ for $\widetilde{w}_2 = 0$,
- $(r, a, \delta) = (2, 2, 1)$ for $(\widetilde{w}_2 \cap \widetilde{w}_2) = 2 \bmod 4$,
- $(r, a, \delta) = (2, 2, 0)$ for $(\widetilde{w}_2 \cap \widetilde{w}_2) = 0 \bmod 4$.

As we have already described in detail earlier, these three theories are in turn related to F-theory on Calabi-Yau three folds on base F_4, F_1 and $F_0(\equiv CP^1 \times CP^1)$ respectively.

The results described in this talk, when combined with the other known duality symmetries of string theory, leads us to the duality cycle displayed in Fig.3 which was first guessed by Gimon and Johnson[12]. There exists independent understanding of the duality involving each link in this cycle. In this talk we discussed two of these links, the ones connecting the topmost box with the two boxes on the right hand side of the diagram. Duality between F-theory on Calabi-Yau with base F_n and $E_8 \times E_8$ heterotic string theory on K3 with instanton number $(12 - n)$ in the first E_8 and $(12 + n)$ in the second E_8 was discussed in ref.[6]. The equivalence between $E_8 \times E_8$ theory on $K3$ and $Spin(32)/Z_2$ hertertotic theory on $K3$ was established in refs.[10, 11]. Finally the horizontal link in the lower part of the diagram follows from the conjectured duality between type I and SO(32) heterotic string theory in ten dimensions[13].

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